Example: a, b, c, d=6, 5, 3, 4, respectively, in (9) gives

$$y^3-86y^2+894y-1216=0.$$

By Horner's method, $y=75.7270176+$ Diagonal $2x=6.1533+$ agreeing with a close drawing.

[Mr. Bell sent us this solution March 14, 1895. We have looked it over carefully and believe that it is entirely correct. The solution published in the July-August number of Vol. II is of a particular case. EDITOR.]

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

57. Proposed by F. M. McGAW, A. M., Professor of Mathematics in Bordentown Military Institute, Bordentown, New Jersey.

Solve the following equation: $(1+x^2)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0.$

I. Solution by WILLIAM E. HEAL, A. M., Member of the London Mathematical Society, and Treasurer of Grant County, Marion, Indiana.

Let $y=bx\left[\int zdx+c\right]$ and the equation becomes

$$x(1+x^2)\frac{dz}{dx}+2z=0, \text{ or } \frac{dz}{z}+\frac{2dx}{x(1+x^2)}=0.$$

$$\therefore z=c'(1+[1/x^2])\,;\ y=bx\{c'(x-[1/x])+c\},\ =Bx+A(1-x^2).$$

II. Solution by H. C. WHITAKER, M. E., Ph. D., Professor of Mathematics in Manual Training School, Philadelphia, Pennsylvania.

Proceeding to obtain a solution in series, both values of y are found to terminate immediately. The complete primitive is $y=Ax+B(x^2-1)$.

III. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

It is shown (*Forsyth's Differential Equations*, Article 58) that

$$d^2y/dx^2+P(dy/dx)+Qy=R\ \ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

gives, when $y=vw$ .... (2).

$$w\frac{d^2v}{dx^2}+(2\frac{dw}{dx}+Pw)\frac{dv}{dx}+(\frac{d^2w}{dx^2}+P\frac{dw}{dx}+Qw)v=R \ldots\ldots\ldots\ldots\ldots\ldots (3),$$

with the conditional equations:

$$\frac{d^2w}{dx^2}+P\frac{dw}{dx}+Qw=0 \ldots\ldots\ldots\ldots\ldots\ldots (4),$$

$$\frac{d^2v}{dx^2}+[(2/w)\frac{dw}{dx}+P]dv/dx=R/w \ldots\ldots\ldots\ldots (5).$$

w being supposed known from (4) gives

$$w^2\frac{dv}{dx}\varepsilon^{-\int Pdx}=A+\int wR\varepsilon^{\int Pdx}dx \ldots\ldots\ldots\ldots (6),$$

and $v=B+A\int\frac{dx}{w^2}\varepsilon^{-\int Pdx}+\int\frac{dx}{w^2}\varepsilon^{-\int Pdx}\int wR\varepsilon^{\int Pdx}dx \ldots\ldots\ldots\ldots (7).$

Now (4) is of the same form as (1) excepting that the right member is 0; so that if we have a solution of (4) we have that of (1) when $R=0$.

The given equation is

$$\frac{d^2y}{dx^2}-\frac{2x}{1+x^2}\frac{dy}{dx}+\frac{2}{1+x^2}y=0 \ldots\ldots\ldots\ldots (8);$$

then $P=-2x/(1+x^2)$, and a particular solution is

$y=x \ldots\ldots\ldots\ldots (9)$, or $w=x \ldots\ldots\ldots\ldots (10)$.

Then (7) gives $v=B--A\int\left(\frac{dx}{x^2}+1\right)=B+A(x-[x/1])$,

and $y=vx=Bx+A(x^2-1)$, the required solution.

[As will be seen from the last solution both forms are correct. The first form is given as the answer, on page 336 of *Byerly's Integral Calculus*. EDITOR.]

Also solved by *O. W. ANTHONY*, *W. C. M. BLACK*, *J. SCHEFFER*, *G. B. M. ZERR*. and *P. S. BERG*.

58. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics in Columbian University, Washington, D. C.

A line passes through a fixed point and rotates uniformly about this point. Another line passes through a point which moves uniformly along the arc of a given curve and rotates uniformly about this point. Develop a method for finding the locus of intersection of these two lines. Apply to case of circle and straight line.